

Superdeformed structures and low Ω parity doublet in Ne–S nuclei near neutron drip-line

Shailesh K. Singh, S.K. Patra and C.R. Praharaj
Institute of Physics, Bhubaneswar-751 005, India
 (Dated: January 27, 2014)

The structure of Ne, Na, Mg, Al, Si, P and S nuclei near the neutron drip-line region is investigated in the frame-work of relativistic mean field theory and non-relativistic Skyrme Hartree-Fock formalism. The recently discovered nuclei ^{40}Mg and ^{42}Al , which are beyond the drip-line predicted by various mass formulae are located within these models. We find many largely deformed neutron-rich nuclei, whose structures are analyzed. From the structure anatomy, we find that at large deformation, low Ω orbits of opposite parities (e.g. $\frac{1}{2}^+$ and $\frac{1}{2}^-$) occur close to each other in energy.

PACS numbers: 21.10.-k, 21.10.dr, 21.10.Ft, 21.30.-x, 24.10.-1, 24.10.Jv

I. INTRODUCTION

The structure of light nuclei near the neutron drip-line is one of the interesting topic for a good number of exotic phenomena. Nuclei in this region are quite different in collectivity and clustering features than the stable counterpart in the nuclear chart. For example, the neutron magic property is lost for $N = 8$ in ^{12}Be [1] and $N = 20$ in ^{32}Mg [2]. The unexpectedly large reaction cross-section for ^{22}C gives the indication of neutron halo structure [3]. The discovery of large collectivity of ^{34}Mg by Iwasaki et al. [4] is another example of such exotic properties. The deformed structures, core excitation and the location of drip-line for Mg and neighboring nuclei are few of the interesting properties of investigation. In this context, the discovery of ^{40}Mg and ^{42}Al , once predicted to be nuclei beyond the drip-line by various mass formulae [5, 6] show to need the modification of the mass models.

On the other hand, the appearance of $N = 16$ as magic number in ^{24}O and the existence of neutron halo in ^{11}Li are established observations [7]. However, the proposed proton [8] (^8B) and neutron [9, 10] halo (^{14}Be , ^{17}B , ^{31}Ne) in the exotic nuclei are currently under investigations. In addition to these, the cluster structure of entire the light mass nuclei and the skin formation in neutron-drip isotopes motivate us for the study of light mass drip-line nuclei. In this paper, our aim is to study the neutron drip-line for Ne–S isotopic chain in the frame-work of a relativistic mean field (RMF) and nonrelativistic Skyrme Hartree-Fock (SHF) formalisms and analyzed the features of large quadrupole deformation of these isotopes.

The paper is organized as follows: The RMF and SHF formalisms are described briefly in Section II. The results obtained from our calculations are discussed in Section III. Finally, summary and concluding remarks are given in Section IV.

II. THE FORMALISM

The mean field methods like SHF and RMF have been widely used in the study of binding energies, root mean square radii, quadrupole deformation and other bulk properties of nuclei [11, 12]. In general, one can say that although the older parametrizations of SHF and RMF have some limitation to

predict the experimental observables, the recent forces are good enough to reproduce the bulk properties not only near the β -stability line but also far away from it. Here, we use these two successful models [11–27] to learn about the properties of drip-line nuclei Ne–S.

A. The Skyrme Hartree-Fock (SHF) Method

The general form of the Skyrme effective interaction used in the mean-field model can be expressed as a Hamiltonian density \mathcal{H} [13–19]. This \mathcal{H} is written expressed as a function of some empirical parameters given as:

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{eff} + \dots, \quad (1)$$

where \mathcal{K} is the kinetic energy term, \mathcal{H}_0 the zero range, \mathcal{H}_3 the density dependent and \mathcal{H}_{eff} the effective-mass dependent terms, which are relevant for calculating the properties of nuclear matter. These are functions of 9 parameters t_i , x_i ($i = 0, 1, 2, 3$) and η , and are given as

$$\mathcal{H}_0 = \frac{1}{4}t_0 [(2 + x_0)\rho^2 - (2x_0 + 1)(\rho_p^2 + \rho_n^2)], \quad (2)$$

$$\mathcal{H}_3 = \frac{1}{24}t_3\rho^\eta [(2 + x_3)\rho^2 - (2x_3 + 1)(\rho_p^2 + \rho_n^2)], \quad (3)$$

$$\mathcal{H}_{eff} = \frac{1}{8}[t_1(2 + x_1) + t_2(2 + x_2)]\tau\rho + \frac{1}{8}[t_2(2x_2 + 1) - t_1(2x_1 + 1)](\tau_p\rho_p + \tau_n\rho_n) \quad (4)$$

The kinetic energy $\mathcal{K} = \frac{\hbar^2}{2M}\tau$, a form used in the Fermi gas model for non-interacting Fermions. The other terms, representing the surface contributions of a finite nucleus with b_4 and b'_4 as additional parameter, are

$$\begin{aligned} \mathcal{H}_{S\rho} = & \frac{1}{16} \left[3t_1 \left(1 + \frac{1}{2}x_1 \right) - t_2 \left(1 + \frac{1}{2}x_2 \right) \right] (\vec{\nabla}\rho)^2 \\ & - \frac{1}{16} \left[3t_1 \left(x_1 + \frac{1}{2} \right) + t_2 \left(x_2 + \frac{1}{2} \right) \right] \\ & \times \left[(\vec{\nabla}\rho_n)^2 + (\vec{\nabla}\rho_p)^2 \right], \end{aligned} \quad (5)$$

$$\mathcal{H}_{S\vec{J}} = -\frac{1}{2} \left[b_4\rho\vec{\nabla} \cdot \vec{J} + b'_4(\rho_n\vec{\nabla} \cdot \vec{J}_n + \rho_p\vec{\nabla} \cdot \vec{J}_p) \right]. \quad (6)$$

Here, the total nucleon number density $\rho = \rho_n + \rho_p$, the kinetic energy density $\tau = \tau_n + \tau_p$, and the spin-orbit density $\vec{J} = \vec{J}_n + \vec{J}_p$. The subscripts n and p refer to neutron and proton, respectively. The nucleon mass is represented by m . The $\vec{J}_q = 0$, $q = n$ or p , for spin-saturated nuclei, i.e., for nuclei with major oscillator shells completely filled or empty. The total binding energy (BE) of a nucleus is the integral of \mathcal{H} .

B. The Relativistic Mean Field (RMF) Method

The relativistic mean field approach is well-known and the theory is well documented [20–25]. Here we start with the relativistic Lagrangian density for a nucleon-meson many-body system as:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_i \{ i\gamma^\mu \partial_\mu - M \} \psi_i + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 \\ & - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - g_s \bar{\psi}_i \psi_i \sigma - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} \\ & + \frac{1}{2} m_w^2 V^\mu V_\mu - g_w \bar{\psi}_i \gamma^\mu \psi_i V_\mu \\ & - \frac{1}{4} \vec{B}^{\mu\nu} \cdot \vec{B}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{R}^\mu \cdot \vec{R}_\mu - g_\rho \bar{\psi}_i \gamma^\mu \vec{\tau} \psi_i \cdot \vec{R}_\mu \\ & - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e \bar{\psi}_i \gamma^\mu \frac{(1 - \tau_{3i})}{2} \psi_i A_\mu. \end{aligned} \quad (7)$$

All the quantities have their usual meanings. From the relativistic Lagrangian, we obtain the field equations for the nucleons and mesons. These equations are solved by expanding the upper and lower components of the Dirac spinor and the boson fields in an axially deformed harmonic oscillator basis. The set of coupled equations is solved numerically by a self-consistent iteration method. The total energy of the system in RMF formalism is given by

$$E_{total} = E_{part} + E_\sigma + E_\omega + E_\rho + E_c + E_{pair} + E_{c.m.}, \quad (8)$$

where E_{part} is the sum of the single particle energies of the nucleons and E_σ , E_ω , E_ρ , E_c , E_{pair} , E_{cm} are the contributions of the meson fields, the Coulomb field, pairing energy and the center-of-mass energy, respectively.

C. Pairing correlation

To take care of the pairing correlation for open shell nuclei, the constant gap BCS-approach is used in our calculations. The pairing energy expression is written as:

$$E_{pair} = -G \left[\sum_{i>0} u_i v_i \right]^2, \quad (9)$$

with G =pairing force constant, v_i^2 and $u_i^2 = 1 - v_i^2$ are the occupation probabilities [28, 29]. The variational approach with respect to v_i^2 gives the BCS equation [28]

$$2\epsilon_i u_i v_i - \Delta(u_i^2 - v_i^2) = 0, \quad (10)$$

using $\Delta = G \sum_{i>0} u_i v_i$. The occupation number is defined as:

$$n_i = v_i^2 = \frac{1}{2} \left[1 - \frac{\epsilon_i - \lambda}{\sqrt{(\epsilon_i - \lambda)^2 + \Delta^2}} \right]. \quad (11)$$

The chemical potentials λ_n and λ_p are determined by the particle numbers for neutrons and protons. The pairing energy is computed as $E_{pair} = -\Delta \sum_{i>0} u_i v_i$. For a particular value of Δ and G , the pairing energy E_{pair} diverges, if it is extended to an infinite configuration space. In fact, in all realistic calculations with finite range forces, the contribution of states of large momenta above the Fermi surface (for a particular nucleus) to Δ decreases with energy. We use a pairing window, where the equations are extended up to the level $|\epsilon_i - \lambda| \leq 2(41A^{-1/3})$. The factor 2 has been determined so as to reproduce the pairing correlation energy for neutrons in ^{118}Sn using Gogny force [20, 29, 30]. The values of Δ_n and Δ_p are taken from [31], as input in the BCS-equation.

We compare the results with various simple and sophisticated pairing prescriptions like BCS-delta force [32] and BCS density dependent delta force [33]. These calculations have been done only for ^{20}Ne and ^{47}Al nucleus in both SkI4 and NL3 force parameter sets. We have given these results in Table I along with experimental results like quadrupole deformation parameter β_2 [34], total binding energy (BE) [35] and root mean square charge radius (r_{ch}) [36]. We find that, for this lighter mass region of the periodic chart, pairing is less important for majority of the cases. With pairing, the deformation becomes negligible for ^{20}Ne and we do not get the experimental deformation parameter in RMF calculations. With no pairing, we reproduce substantially the deformation parameter in RMF because the *density of states* near Fermi surface for such light nuclei are small and not conducive to pairing [37]. To understand the influence of pairing on the open shell nuclei, we have taken into account the experimental data, wherever available. The SHF(SkI4) results are used as guideline in the absence of these data. We realized after comparing the calculated β_2 of RMF and SHF with experimental data that the quadrupole deformation of SHF is closer to experiment without taking pairing correlation into account. For example, when we use the Δ_n and Δ_p from the experimental binding energy of odd-even values or from the empirical formula of Ref. [31, 38] to calculate β_2 for $^{20,22,24,26,28}\text{Ne}$ in RMF(NL3), we find $\beta_2 \sim 0.18, 0.35, 0.19, 0.0, 0.0$, respectively for these isotopes agreeing with the result of Lalazissis et al [39]. These β_2 strongly disagree with the measured values ($\beta_2(expt.) = 0.723, 0.562, 0.45, 0.498, 0.50$) [34]. Similar effects are also seen in other considered isotopes. On the other hand, if we ignore pairing, then the calculated results are often better and these β_2 are quite close to the experimental data. The influence of pairing is also visible in the total binding energy. In some of the cases, even a couple of MeV difference in total binding energy is found with and without taking pairing correlation into account in RMF formalism. Contrary to the RMF, the pairing in the SHF formalism is almost insensitive to quadrupole deformation for the considered mass region. Thus, we have performed the calculations through out the paper without considering pairing into account.

TABLE I: Calculation of binding energy (BE), quadrupole deformation parameter β_2 , root mean square of matter radius (r_{rms}) and charge radius (r_{ch}) by taking various pairing methods. We have given these results for both SkI4 and NL3 parameter sets with experimental data [34–36].

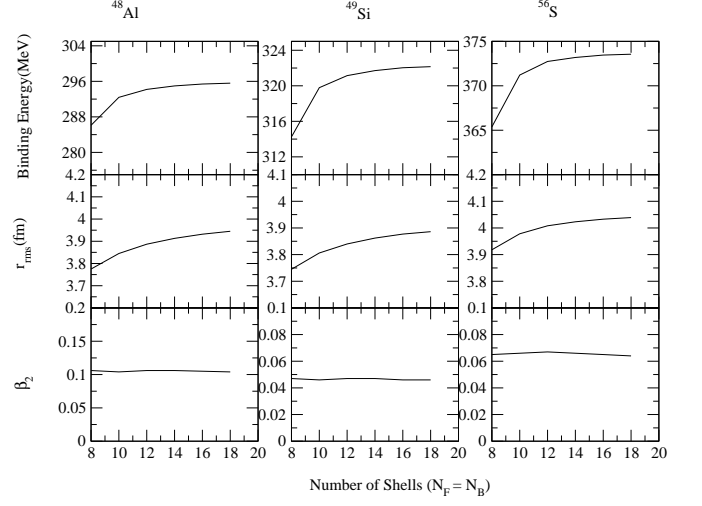
SkI4					
Nucl.	Type of Pairing	β_2	r_{rms}	BE	r_{ch}
^{20}Ne	No pairing	0.549	2.911	156.8	3.030
	BCS-delta force	0.548	2.910	156.8	3.030
	BCS-dens.dep.delta-force	0.548	2.910	156.8	3.030
^{47}Al	No pairing	0.006	3.972	287.8	3.324
	BCS-delta force	0.007	3.957	288.7	3.317
	BCS-dens.dep.delta-force	0.055	3.970	288.0	3.322
NL3					
^{20}Ne	No pairing	0.537	2.846	156.7	2.972
	BCS-delta	0.036	2.920	154.9	3.055
^{47}Al	No pairing	0.090	3.832	294.6	3.246
	BCS-delta	0.081	3.834	294.8	3.246
Exp. Results					
^{20}Ne		0.728		160.6	3.005
^{47}Al				—	

D. Pauli Blocking and Harmonic oscillator basis

For even-even nucleus $\pm m$ orbits are pairwise occupied and the mean field has time reversal symmetry. But in the case of odd nucleon the time reversal symmetry is broken. To take care of the odd nucleon, we employ the blocking method [40]. We put the last nucleon in one of the conjugate states $\pm m$ and keeping other state empty. In this way we follow the time reversal symmetry for odd-even and odd-odd nuclei. We repeat this calculation by putting the odd nucleon in all the near by state of the conjugate level to determine the maximum binding energy of the ground state [40, 41].

In our present calculations the nuclei are treated as axially-symmetrically deformed, with z -axis as the symmetric axis. Spherical symmetry is no longer present in general and therefore j is not a good quantum number any more. Because of axial symmetry, each orbit is denoted by the quantum number m of J_z and is a superposition of $|jm\rangle$ states with various j values. The densities are invariant with respect to a rotation around the symmetry axis. For numerical calculations, the wavefunctions are expanded in a deformed harmonic oscillator potential basis and solved self-consistently in an iteration method. The major oscillator quanta for Fermion N_F and bosons N_B are taken as $N_{max} = 12$. The convergence of our numerical results are tested in Fig. 1 for BE, matter radius r_{rms} and quadrupole deformation parameter for some selected nuclei like ^{48}Al , ^{49}Si and ^{56}S . Here, the results are estimated from $N_F = N_B = 8$ to $N_F = N_B = 18$, which are shown in Fig. 1. From this analysis, we observed that the β_2 values almost identical with the variation of oscillator quanta. However, the rms radii and binding energy vary till $N_F = N_B = 12$, beyond which the results are unchanged. It is well known that harmonic oscillator basis is not suitable in dripline nuclei due to the asymptotic behavior of the density distribution. To resolve this problem, efforts have been made for solving the equations in coordinate space [42–44].

FIG. 1: The change in binding energy BE, root mean square matter radius (r_{rms}), quadrupole deformation parameter β_2 with Fermionic N_F and bosonic N_B harmonic oscillator basis for some selected nuclei.



Some other kinds of basis like transformed harmonic oscillator basis [45], Gaussian expansion method [46] and Woods-Saxon basis [47, 48] are also available in literature. The inclusion of sufficiently large harmonic oscillator model space gives reasonably convergent results. This type of prescription is already done in Ref. [49]. However, to fully include continuum effects more work has to be done (by use of basis of finite potentials and inclusion of correlation effects in Hartree-Bogoliubov scheme [50]).

E. Ground state properties from the SHF and RMF models

Certainly for light mass nuclei, the correction of centre of mass motion can not be ignored and it should be done self-consistently. That means, in the evaluation of centre-of-mass energy, one should evaluate $E_{CM} = \frac{\langle F | P^2 | F \rangle}{2M}$ using $|F\rangle = |F\rangle_{RMF}$ wavefunction. In this case, one has to calculate the matrix elements directly. However, this procedure is more involved and in the present calculations we have subtracted the spurious centre-of-mass motion using the Elliott-Skyrme approximation and the approximate analytical expression is written as $E_{CM} = \frac{3}{4} A^{-1/3} \text{ MeV}$ (harmonic oscillator approximation), where A is mass number [51–53] and expect that the two results should not differ drastically. The quadrupole moment deformation parameter β_2 is evaluated from the resulting proton and neutron quadrupole moments through:

$$Q = Q_n + Q_p = \sqrt{\frac{16\pi}{5}} \left(\frac{3}{4\pi} A R^2 \beta_2 \right), \quad (12)$$

where $R = 1.2 A^{1/3}$. The root mean square radii of protons and matter distribution are defined as $\langle r_p^2 \rangle = \frac{1}{Z} \int \rho_p(r_\perp, z) r^2 d\tau$,

and $\langle r_{rms}^2 \rangle = \frac{1}{A} \int \rho(r_\perp, z) r^2 d\tau$; respectively, where Z is the proton number and $\rho_p(r_\perp, z)$ is the deformed proton and $\rho(r_\perp, z)$ is the total nucleon density distribution. The proton and charge rms radius is connected through the relation $r_{ch} = \sqrt{r_p^2 + 0.64}$ [41].

We use the well known NL3 parameter set [54] for the RMF formalism. This set not only reproduces the properties of stable nuclei but also well predicts for those far from the β -stability valley. Also, the isoscalar monopole energy agrees excellently with the experimental values for different regions of the Periodic Table. The measured superdeformed minimum in ^{194}Hg is 6.02 MeV above the ground state, whereas in RMF calculation with NL3 set, this number is 5.99 MeV [54].

For SHF model, we use the Skyrme SkI4 set with $b_4 \neq b'_4$ [26]. This parameter set is designed for considerations of proper spin-orbit interaction in finite nuclei, related to the isotope shifts in Pb region and is better suited for the study of exotic nuclei. Several more recent Skyrme parameters such as SLy1-10, SkX, SkI5 and SkI6 are obtained by fitting the Hartree-Fock (HF) results with experimental data for nuclei starting from the valley of stability to neutron and proton drip-lines [13, 26, 27, 55].

III. RESULTS AND DISCUSSIONS

The binding energy BE, rms charge radius r_{ch} and quadrupole deformation parameter β_2 of the isotopes of Ne, Na, Mg, Al, Si, P and S are calculated near the drip-line region. For this, both the relativistic and non-relativistic models are used.

A. Binding energy and neutron drip-line

The ground state binding energy (BE) for Ne, Na, Mg, Al, Si, P and S isotopes are selected by comparing the binding energy obtained from the prolate, oblate and spherical solutions for a particular nucleus. For a given nucleus, the maximum binding energy corresponds to the ground state and other solutions are obtained as various excited intrinsic states. In Table II, the ground state binding energy for the heaviest known isotopes for the discussed nuclei are compared with the experimental data [35]. The binding energy for ^{31}Ne is 216.0 MeV with RMF (NL3) and these are 213.2 and 211.4 MeV in SHF(SkI4) and experiment, respectively. Similarly, these results for ^{45}S respectively are 353.4, 350.4 and 354.7 MeV in RMF, SHF and experiment. Analyzing the data of Table II, generally one finds that BE of RMF is slightly over estimated and SHF is underestimated than the experimental values. However, the overall agreement of the calculated energies are within an acceptable range with the experimental data.

We have listed the neutron drip-lines in Table III, which are obtained from the ground state binding energy for neutron rich Ne, Na, Mg, Al, Si, P and S nuclei. The drip-line is determined by setting the condition that the minimum value of two neutron separation energy $S_{2n} = BE(N, Z) - BE(N -$

TABLE II: The calculated ground state binding energy obtained from SHF and RMF theory are compared with the experimentally known heaviest isotope for Ne, Na, Mg, Al, Si, P and S [35].

Nucl.	RMF	SHF	Expt.	Nucleus	RMF	SHF	Expt.
^{31}Ne	216.0	213.2	211.4	^{32}Na	234.5	233.4	230.9
^{36}Mg	263.9	260.2	260.8	^{38}Al	283.5	281.4	280.3
^{41}Si	310.1	307.2	307.9	^{43}P	331.7	329.0	330.7
^{45}S	353.4	350.4	354.7				

TABLE III: The predicted mass number of neutron drip-line for Ne, Na, Mg, Al, Si, P and S nucleus in RMF (NL3) and SHF (SkI4) parameter sets are compared with infinite nuclear matter (INM) mass model [56], finite range droplet model (FRDM) [57] and the nuclei with the largest neutron numbers so far experimentally detected [35] along with the number shown in parenthesis are the experimentally extrapolated values.

Nucl.	RMF	SHF	INM	FRDM	Expt.
Ne	34	34	34	33	31 (34)
Na	40	37	37	36	32 (37)
Mg	40	40	39	40	36 (40)
Al	48	48	42	42	38 (43)
Si	54	48	45	43	41 (45)
P	54	55	49	48	43 (47)
S	55	55	51	51	45 (49)

$2, Z) \geq 0$. The nuclei with the largest neutron numbers so far experimentally detected in an isotopic chain along with the extrapolated data are also displayed in the last column of Table III. The numbers given in the parenthesis are the experimentally extrapolated values [35]. To get a qualitative understanding of the prediction of neutron drip-line, we have compared our results with the infinite nuclear matter (INM) [56] and finite range droplet model (FRDM) [57] mass estimation. The RMF and SHF drip-lines coincide with each other for Ne, Mg, Al and S. In case of Na and Si the RMF drip nuclei are found to be 3 and 6 unit heavier than the SHF prediction. The INM prediction of drip nuclei are always in the heavier side than FRDM. From Table III, we find that the experimental effort has almost reached to the INM and FRDM prediction of drip nuclei for lighter mass region.

The theoretical prediction of drip nuclei are very important after the discovery of ^{40}Mg and ^{42}Al [5]. These two nuclei are considered to be beyond the drip-line (neutron-unbound) in some of the mass calculations [6, 58]. The discovery of these two isotopes, suggests the existence of drip-line somewhere in the heavier side. Thus, the study of these isotopes is beyond the scope of the existing mass models [6, 58]. In the present RMF/SHF calculations, the newly discovered ^{40}Mg and ^{42}Al are well within the drip-line. Also, a point of caution, it may be possible that if we allow triaxial deformation in the calculation, then we may get one minimum as a saddle point and another one as triaxial minimum. But this calculation is out of scope in our paper, we are dealing with axial deformed code by using NL3 and SkI4 parameter sets where mostly we find similar results in both the formalisms. This type of prescriptions are used in many of the earlier publica-

TABLE IV: The calculated value of charge radius (r_{ch}), quadrupole moment deformation parameter β_2 and binding energy (BE) for Ne, Na and Mg nuclei in RMF (NL3) and SHF (SkI4) formalisms. We compare our results with experimental β_2 [34], ground state binding energy BE (MeV) [35] and the charge radius r_{ch} (fm) [36].

Nucl.	RMF (NL3)			SHF (SkI4)			Exp.		
	r_{ch}	β_2	BE	r_{ch}	β_2	BE	r_{ch}	β_2	BE
²⁰ Ne	2.970	0.535	156.7	3.030	0.550	156.8	3.006	0.727	160.6
²¹ Ne	2.953	0.516	165.9	3.012	0.529	166.8	2.970		167.4
²² Ne	2.940	0.502	175.7	3.010	0.520	175.8	2.953	0.562	177.8
²³ Ne	2.913	0.386	181.8	2.975	0.382	182.2	2.910		183.0
²⁴ Ne	2.881	-0.259	189.1	2.950	-0.250	188.5	2.901	0.45	191.8
²⁵ Ne	2.907	0.272	194.2	2.948	0.170	194.2	2.932		196.0
²⁶ Ne	2.926	0.277	199.9	2.950	0.120	199.4	2.925	0.498	201.6
²⁷ Ne	2.945	0.247	203.9	2.987	0.159	203.2			203.1
²⁸ Ne	2.965	0.225	208.2	3.010	0.160	206.5	2.964	0.50	206.9
²⁹ Ne	2.981	0.161	211.2	3.027	0.010	210.1			207.8
³⁰ Ne	2.998	0.100	215.0	3.050	0.000	213.7			211.3
³¹ Ne	3.031	0.244	216.0	3.057	0.225	213.2			211.4
³² Ne	3.071	0.373	218.6	3.100	0.380	213.1			
³³ Ne	3.095	0.424	219.5	3.148	0.429	213.5			
³⁴ Ne	3.119	0.473	220.9	3.180	0.490	213.5			
²⁴ Na	2.964	0.379	192.3	3.042	0.411	194.0	2.974		193.5
²⁵ Na	2.937	0.273	200.6	3.024	0.314	201.4	2.977		202.5
²⁶ Na	2.965	0.295	207.1	3.027	0.274	208.4	2.993		208.1
²⁷ Na	2.993	0.323	214.2	3.043	0.282	214.9	3.014		214.8
²⁸ Na	2.993	0.272	219.0	3.058	0.234	219.7	3.040		218.4
²⁹ Na	3.004	0.232	224.3	3.072	0.194	224.3	3.092		222.8
³⁰ Na	3.031	0.169	228.1	3.079	0.030	228.6	3.118		225.1
³¹ Na	3.047	0.108	232.7	3.103	0.000	233.5	3.170		229.3
³² Na	3.077	0.237	234.5	3.121	0.187	233.4			230.9
³³ Na	3.113	0.356	237.9	3.172	0.352	234.9			
³⁴ Na	3.137	0.404	239.8	3.198	0.407	236.2			
³⁵ Na	3.161	0.450	242.3	3.224	0.457	237.4			
³⁶ Na	3.175	0.481	242.5	3.235	0.501	237.5			
³⁷ Na	3.190	0.512	243.1	3.251	0.541	237.6			
³⁸ Na	3.199	0.491	243.4						
³⁹ Na	3.209	0.472	244.1						
⁴⁰ Na	3.228	0.477	243.4						
²⁴ Mg	3.043	0.487	194.3	3.130	0.520	195.2	3.057	0.605	198.3
²⁵ Mg	3.009	0.376	202.9	3.103	0.432	204.3	3.028		205.6
²⁶ Mg	2.978	0.273	212.5	3.080	-0.300	213.2	3.034	0.482	216.7
²⁷ Mg	3.015	0.310	220.2	3.096	0.339	221.5			223.1
²⁸ Mg	3.048	0.345	228.7	3.110	0.340	229.0		0.491	231.6
²⁹ Mg	3.055	0.289	234.3	3.118	0.283	235.0			235.3
³⁰ Mg	3.062	0.241	240.5	3.120	-0.180	240.5		0.431	241.6
³¹ Mg	3.075	0.179	245.1	3.123	0.030	246.1			243.9
³² Mg	3.090	0.119	250.5	3.150	0.000	252.0		0.473	249.7
³³ Mg	3.117	0.233	253.1	3.165	0.155	253.0			252.0
³⁴ Mg	3.150	0.343	257.3	3.210	0.330	255.1			256.7
³⁵ Mg	3.173	0.388	260.5	3.239	0.393	257.8			257.5
³⁶ Mg	3.198	0.432	263.9	3.265	0.440	260.2			260.8
³⁷ Mg	3.212	0.462	264.9	3.279	0.469	261.0			
³⁸ Mg	3.227	0.492	266.3	3.295	0.490	261.6			
³⁹ Mg	3.237	0.473	267.8	3.307	0.485	262.4			
⁴⁰ Mg	3.247	0.456	269.7	3.320	0.470	262.8			

TABLE V: Same as Table IV, for Al and Si isotopes.

Nucl.	RMF			SHF			Exp.		
	r_{ch}	β_2	BE	r_{ch}	β_2	BE	r_{ch}	β_2	BE
²⁴ Al	3.097	0.388	182.3	3.174	0.413	185.0			183.6
²⁵ Al	3.072	0.381	197.7	3.164	0.430	199.5			200.5
²⁶ Al	3.052	-0.275	207.8	3.122	0.315	211.4			211.9
²⁷ Al	3.053	-0.292	221.9	3.092	0.204	222.7	3.061		225.0
²⁸ Al	3.037	-0.208	238.6	3.105	0.202	232.5			232.7
²⁹ Al	3.033	-0.141	245.6	3.126	0.241	241.5			242.1
³⁰ Al	3.070	-0.184	253.8	3.139	0.194	248.7			247.8
³¹ Al	3.101	-0.205	259.8	3.161	-0.192	256.0			255.0
³² Al	3.103	-0.111	261.2	3.162	0.020	262.6			259.2
³³ Al	3.165	-0.333	269.4	3.183	0.000	269.8			264.7
³⁴ Al	3.134	-0.108	275.1	3.198	0.090	271.7			267.3
³⁵ Al	3.167	0.268	274.1	3.229	0.250	274.4			272.5
³⁶ Al	3.173	-0.189	277.7	3.254	0.320	277.4			274.4
³⁷ Al	3.208	0.355	281.5	3.278	0.371	280.1			278.6
³⁸ Al	3.214	-0.254	283.5	3.288	0.378	281.4			280.3
³⁹ Al	3.236	-0.299	286.7	3.383	-0.121	287.1			
⁴⁰ Al	3.257	-0.336	290.4	3.316	0.403	284.2			
⁴¹ Al	3.278	-0.370	290.6	3.338	-0.367	285.9			
⁴² Al	3.281	-0.355	291.2	3.341	-0.339	286.2			
⁴³ Al	3.282	-0.338	292.2	3.341	-0.312	286.6			
⁴⁴ Al	3.274	-0.288	293.6	3.340	-0.282	287.0			
⁴⁵ Al	3.271	-0.263	293.5	3.338	-0.250	287.6			
⁴⁶ Al	3.359	0.341	294.5	3.326	-0.129	287.7			
⁴⁷ Al	3.246	0.090	294.8	3.318	-0.004	288.7			
⁴⁸ Al	3.319	-0.252	294.0	3.347	-0.060	287.6			
²⁸ Si	3.122	-0.331	232.1	3.190	-0.350	233.6	3.122	0.407	236.5
²⁹ Si	3.035	0.001	240.7	3.176	-0.272	243.1	3.118		245.0
³⁰ Si	3.070	0.148	250.6	3.170	-0.210	252.6	3.134	0.315	255.6
³¹ Si	3.108	-0.180	259.1	3.182	-0.199	261.7			262.2
³² Si	3.137	-0.201	268.5	3.200	-0.200	270.5		0.217	271.4
³³ Si	3.131	-0.084	275.6	3.196	0.010	278.1			275.9
³⁴ Si	3.148	0.000	284.4	3.220	0.000	286.3		0.179	283.4
³⁵ Si	3.161	-0.083	287.4	3.226	0.010	289.5			285.9
³⁶ Si	3.186	-0.162	291.5	3.150	0.150	292.4		0.259	292.0
³⁷ Si	3.200	0.238	295.4	3.269	0.247	295.9			294.3
³⁸ Si	3.218	0.281	299.8	3.290	0.310	298.2		0.249	299.9
³⁹ Si	3.224	0.263	302.4	3.298	0.292	301.4			301.5
⁴⁰ Si	3.272	-0.301	306.0	3.310	-0.280	304.0			306.5
⁴¹ Si	3.295	-0.336	310.1	3.349	-0.329	307.2			307.9
⁴² Si	3.318	-0.369	314.6	3.330	-0.350	310.0			
⁴³ Si	3.320	-0.356	315.2	3.377	-0.339	311.1			
⁴⁴ Si	3.322	-0.342	316.2	3.380	-0.300	311.6			
⁴⁵ Si	3.316	-0.308	317.5	3.374	-0.282	312.9			
⁴⁶ Si	3.303	-0.262	319.3	3.370	-0.240	313.5			
⁴⁷ Si	3.345	-0.298	319.8	3.340	0.030	314.3			
⁴⁸ Si	3.263	0.001	321.8	3.350	0.000	315.4			
⁴⁹ Si	3.290	0.045	321.1						
⁵⁰ Si	3.341	-0.159	321.5						
⁵¹ Si	3.358	-0.135	321.2						
⁵² Si	3.371	0.082	321.4						
⁵³ Si	3.391	0.042	321.6						
⁵⁴ Si	3.415	0.000	322.3						

tions [59].

TABLE VI: Same as Table IV, for P and S isotopes.

Nucl.	RMF			SHF			Exp.		
	r_{ch}	β_2	BE	r_{ch}	β_2	BE	r_{ch}	β_2	BE
³⁰ P	3.138	0.130	246.3	3.189	0.026	249.9	3.189		250.6
³¹ P	3.158	0.205	258.3	3.201	0.105	261.1			262.9
³² P	3.174	-0.143	267.1	3.216	0.069	270.9			270.9
³³ P	3.201	-0.183	277.5	3.246	-0.167	280.5			281.0
³⁴ P	3.201	-0.082	285.8	3.248	0.001	289.9			287.2
³⁵ P	3.216	-0.001	295.4	3.265	0.000	299.2			295.6
³⁶ P	3.227	0.120	299.5	3.272	0.007	303.3			299.1
³⁷ P	3.246	0.209	305.0	3.290	0.148	307.4			305.9
³⁸ P	3.260	0.250	310.4	3.313	0.240	311.7			309.6
³⁹ P	3.275	0.288	316.1	3.334	0.301	316.1			315.9
⁴⁰ P	3.281	0.274	320.1	3.343	0.290	319.6			319.2
⁴¹ P	3.288	0.261	324.4	3.355	0.295	322.7			324.2
⁴² P	3.306	0.301	327.3	3.371	0.320	325.6			326.3
⁴³ P	3.346	-0.323	331.7	3.398	-0.320	329.0			330.7
⁴⁴ P	3.346	-0.302	333.3	3.398	-0.293	330.6			
⁴⁵ P	3.315	0.222	335.4	3.397	-0.264	332.4			
⁴⁶ P	3.342	-0.251	337.5	3.397	-0.237	334.2			
⁴⁷ P	3.341	-0.232	340.0	3.399	-0.218	336.0			
⁴⁸ P	3.381	-0.271	341.2	3.379	0.034	337.4			
⁴⁹ P	3.328	0.088	343.2	3.387	0.012	339.3			
⁵⁰ P	3.353	0.101	343.7	3.414	-0.061	339.2			
⁵¹ P	3.397	-0.166	344.7	3.437	0.068	339.4			
⁵² P	3.403	0.109	345.2	3.462	0.079	339.7			
⁵³ P	3.428	0.109	346.3	3.487	0.089	340.1			
⁵⁴ P	3.447	0.074	346.6	3.502	0.016	340.5			
⁵⁵ P	3.468	0.037	347.4	3.525	0.001	341.2			
³³ S	3.241	0.197	275.5	3.276	0.119	278.9	3.285	0.252	280.4
³⁴ S	3.257	-0.168	286.5	3.300	-0.160	289.3			291.8
³⁵ S	3.260	-0.078	295.7	3.300	-0.006	299.6			298.8
³⁶ S	3.273	0.002	306.2	3.310	0.000	309.6			308.7
³⁷ S	3.285	0.152	311.6	3.319	-0.008	315.1			313.0
³⁸ S	3.300	0.228	318.6	3.340	0.210	320.2			321.1
³⁹ S	3.312	0.264	325.3	3.354	0.248	326.5			325.4
⁴⁰ S	3.325	0.299	332.4	3.370	0.300	332.1			333.2
⁴¹ S	3.331	0.287	337.7	3.381	0.294	336.9			337.4
⁴² S	3.338	0.277	343.2	3.390	0.290	341.0			344.1
⁴³ S	3.359	0.318	347.2	3.413	0.326	344.7			346.7
⁴⁴ S	3.381	0.367	351.0	3.440	0.370	348.3			351.8
⁴⁵ S	3.375	0.312	353.4	3.430	0.311	350.4			354.7
⁴⁶ S	3.371	0.258	356.6	3.420	0.250	352.5			
⁴⁷ S	3.385	0.257	358.5	3.428	-0.214	354.8			
⁴⁸ S	3.400	0.259	360.8	3.430	-0.200	356.6			
⁴⁹ S	3.403	0.227	362.9	3.430	0.127	358.8			
⁵⁰ S	3.403	0.189	365.5	3.440	0.120	360.8			
⁵¹ S	3.427	0.188	366.4	3.459	-0.090	361.8			
⁵² S	3.451	0.183	367.6	3.490	-0.140	362.5			
⁵³ S	3.463	0.158	369.1	3.508	-0.113	363.6			
⁵⁴ S	3.477	0.139	371.0	3.530	0.000	364.7			
⁵⁵ S	3.494	0.105	371.4	3.541	0.030	365.4			

B. Neutron configuration

Analyzing the neutron configuration for these exotic nuclei, we notice that, for lighter isotopes of Ne, Na, Mg, Al, Si, P and S the oscillator shell $N_{osc} = 3$ is empty in the $[N_{osc}, n_3, \Lambda]\Omega^\pi$. However, the $N_{osc} = 3$ shell gets occupied

gradually with increase of neutron number. In case of Na, $N_{osc} = 3$ starts filling up at ³³Na with quadrupole moment deformation parameter $\beta_2 = 0.356$ and -0.179 with occupied orbits $[330]\frac{1}{2}^-$ and $[303]\frac{7}{2}^-$, respectively. The filling of $N_{osc} = 3$ goes on increasing for Na with neutron number and it is $[330]\frac{1}{2}^-$, $[310]\frac{1}{2}^-$, $[321]\frac{3}{2}^-$ and $[312]\frac{5}{2}^-$ at $\beta_2 = 0.472$ for ³⁹Na. Again for the oblate solution the occupation is $[301]\frac{1}{2}^-$, $[301]\frac{3}{2}^-$, $[303]\frac{5}{2}^-$ and $[303]\frac{7}{2}^-$ for $\beta_2 = -0.375$ for ³⁹Na. In case of Mg isotopes, even for ^{30,32}Mg, the $N_{osc} = 3$ shell has some occupation for the low-lying excited states near the Fermi surface. For ³⁰Mg (at $\beta_2 = 0.599$ with BE = 237.7 MeV) the $N_{osc} = 3$ orbit is $[330]\frac{7}{2}^-$ and for ³²Mg is $[330]\frac{1}{2}^-$ (BE = 248.8 MeV at $\beta_2 = 0.471$). With the increase of neutron number in Mg and Si isotopic chains, the oscillator shell with $N_{osc} = 3$ gets occupied more and more.

In Tables (4 – 6) the results for the ground state solutions are displayed. Thus, the prolate solutions have more binding than the oblate one for Ne, Na, Mg and S isotopes. In some cases, like ^{24–30}Ne the prolate and oblate solutions are in degenerate states. For example, ²⁴Ne has BE = 188.9 and 189.1 MeV at $\beta_2 = 0.278$ and -0.259 respectively. Contrary to this, the ground state solutions for Al and Si are mostly oblate. For example, ³⁴Al has BE = 269.9 and 275.1 MeV at $\beta_2 = 0.159$ and -0.108 respectively. In such cases, the prolate solutions are in low-lying excited intrinsic state. Note that in many cases, there exist low laying superdeformed states.

It is important to list some of the limitations of the results due to the input parameters, mostly comes from E_{pair} and E_{cm} energies. As one can see from Fig. 3, in many cases there are solutions of different shapes lying only a few MeV higher, sometimes even degenerate with the ground states. Such a few MeV difference is within the uncertainty of the predicted binding energies. A slight change in the pairing parameter, among others, may alter the prediction for the ground state shape. With few MeV uncertainty in ground state binding energies, by reassigning the ground state configurations, the deformation may change completely, and make the predictions close to each other and agree with the FRDM predictions as well.

C. Quadrupole deformation

The ground and low-lying excited state deformation systematics for some of the representative nuclei for Ne, Na, Mg, Al, Si, P and S are analyzed. In Fig. 2, the ground state quadrupole deformation parameter β_2 is shown as a function of mass number for Ne, Na, Mg, Al, Si, P and S. The β_2 value goes on increasing with mass number for Ne, Na and Mg isotopes near the drip-line. The calculated quadrupole deformation parameter β_2 for ³⁴Mg is 0.59 which compares well with the recent experimental measurement of Iwasaki et al [4] ($\beta_2 = 0.58 \pm 0.06$). It found that this superdeformed state is 3.2 MeV above than the ground band. Again, the magnitude of β_2 for the drip nuclei reduces with neutron number N and again increases. A region of maximum deformation is found for almost all the nuclei as shown in the figure. It so happens

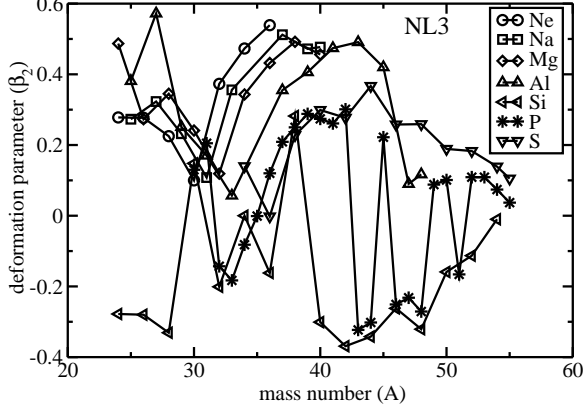


FIG. 2: The ground state quadrupole deformation parameter β_2 versus mass number A for Ne, Na, Mg, Al, Si, P and S isotopes near the drip-line with NL3 parameter set.

in cases like, Ne, Na, Mg and Al that the isotopes are maximum deformed which may be comparable to superdeformed near the drip-line. For Al and Si isotopes, in general, we find oblate solutions in the ground configurations (see Table 5). In many of the cases, the low-lying superdeformed configuration are clearly visible and some of them can be seen in Fig. 2.

D. Shape coexistence

One of the most interesting phenomena in nuclear structure physics is the shape coexistence [59–62]. In some cases of the nuclei, considered near the drip-line, the ground state configuration accompanies a low-lying excited state. In a few cases, it so happens that these two solutions are almost degenerate in energy. For example, in the RMF calculation, the ground state binding energy of ^{24}Ne is 189.1 MeV with $\beta_2 = -0.259$ and the binding energy of the excited low-lying configuration at $\beta_2 = 0.278$ is 188.9 MeV. The difference in BE of these two solutions is only 0.179 MeV. Similarly the solution of prolate-oblate binding energy difference in SkI4 is 0.186 MeV for ^{30}Mg with $\beta_2 = -0.183$ and 0.202. This type of degenerate solutions are observed in most of the isotopes near the drip-line. It is worthy to mention that in the truncation of the basis space, an uncertainty of ≤ 1 MeV in total binding energy may occur. However, this uncertainty in convergence does not effect to determine the shape co-existence, because both the solutions are obtained by using the same model space of $N_F = N_B = 12$.

To show in a quantitative way, we have plotted the prolate-oblate binding energy difference ($BE_p - BE_o$) in Fig. 3. The left hand side of the figure is for relativistic and the right side is the nonrelativistic results. From the figure, it is clear that an island of shape coexistence isotopes are available for Mg and Si isotopes. These shape coexistence solutions are predicted taking into account the intrinsic binding energy. However, the

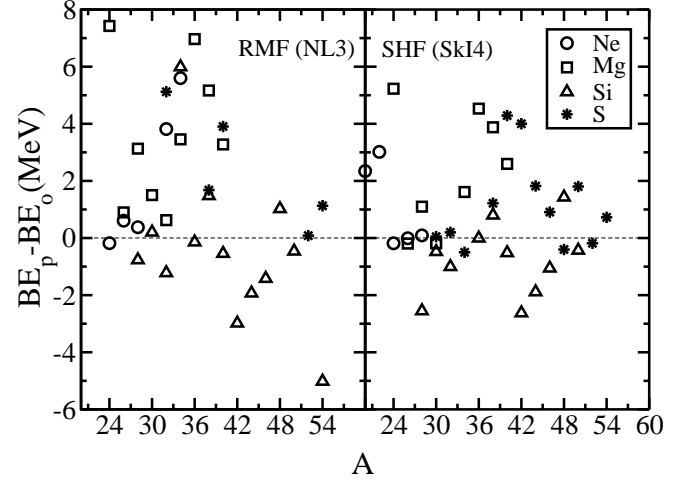


FIG. 3: The difference in binding energy between the prolate-oblate solutions is shown for even-even Ne, Mg, Si and S isotopes near the neutron drip-line with NL3 and SkI4 parameter sets.

actual quantitative energy difference of ground and excited configuration can be given by performing configuration mixing (mixing such as in the generator coordinate method(GCM) [63]) after the angular momentum projection [60].

E. Two neutron separation energy (S_{2n})

The appearance of new and the disappearance of known magic number near the neutron drip-line is a well discussed topic currently in nuclear structure physics [7, 64]. Some of the calculations in recent past predicted the disappearance of the known magic number $N = 28$ for the drip-line isotopes of Mg and S [65–67]. However, magic number 20 retains its magic properties even for the drip-line region. In one of our earlier publications, [68] we analyzed the spherical shell gap at $N = 28$ in ^{44}S and its neighboring ^{40}Mg and ^{42}Si using NL-SH [69] and TM2 parameter sets [53]. The spherical shell gap at $N = 28$ in ^{44}S was found to be intact for the TM2 and is broken for NL-SH parametrization. Here, we plot the two-neutron separation energy S_{2n} for Ne, Mg, Si and S for the even-even nuclei near the drip-line (Fig. 4). The known magic number $N = 28$ is noticed to be absent in ^{44}S . On the other hand, the appearance of a sudden decrease in S_{2n} energy at $N = 34$ in SHF result is quite prominent, which is not clearly visible in RMF prediction. This is just two units ahead than the experimental shell closure at $N = 32$ [70].

F. Superdeformation and Low Ω parity doublets

The deformation-driving $m = \frac{1}{2}^-$ orbits come down in energy in superdeformed solutions from the shell above, in con-

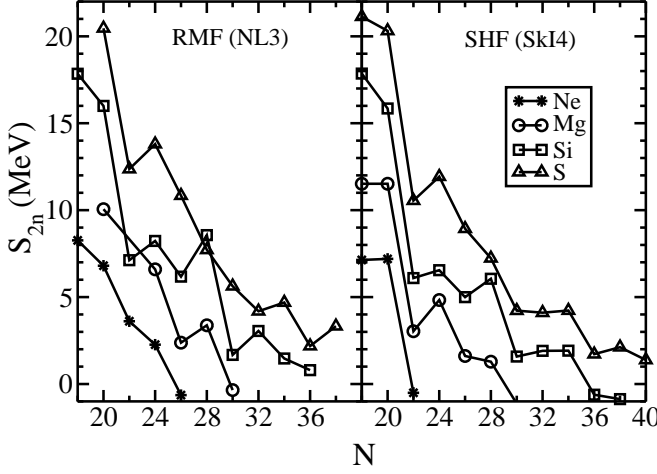


FIG. 4: The two-neutron separation S_{2n} energy versus neutron number N for neutron-rich Ne, Mg, Si and S isotopes.

TABLE VII: Occupation of neutron orbits m^π in ^{47}Al and ^{46}Al driven by the deformation.

A	β_2	$n_{\frac{1}{2}^+}$	$n_{\frac{1}{2}^-}$	$n_{\frac{3}{2}^+}$	$n_{\frac{3}{2}^-}$	$n_{\frac{5}{2}^+}$	$n_{\frac{5}{2}^-}$	$n_{\frac{7}{2}^+}$	$n_{\frac{7}{2}^-}$	$n_{\frac{9}{2}^+}$	$n_{\frac{9}{2}^-}$
^{47}Al	0.09	8	10	4	6	2	2	0	2	0	0
^{47}Al	0.672	10	10	4	6	2	2	0	0	0	0
^{46}Al	0.109	8	9	4	6	2	2	0	2	0	0
^{46}Al	0.701	10	10	4	5	2	2	0	0	0	0

trast to the normal deformed solutions. The occurrence of approximate $\frac{1}{2}^+$, $\frac{1}{2}^-$ parity doublets (degeneracy of $|m|^\pi = \frac{1}{2}^+$, $\frac{1}{2}^-$ states) for the superdeformed solutions are clearly seen in Figs. 5 and 6 where excited superdeformed configurations for ^{32}Mg , ^{34}Mg and for ^{46}Al , ^{47}Al are given (RMF solutions). For each nucleus, we have compared the normal deformed ($\beta_2 \sim 0.1 - 0.3$) and the superdeformed configurations and analyzed the deformed orbits. The $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states for the single particle levels are shown in Fig. 5 (for ^{32}Mg and ^{34}Mg) and Fig. 6 for ^{47}Al and ^{46}Al . The occupation of neutron states (denoted by m^π) in ^{47}Al and ^{46}Al is given in Table VII. In both ^{47}Al and ^{46}Al two neutrons occupying oblate driving $f_{7/2} m = \frac{7}{2}$ orbits in normal deformation are unoccupied in the superdeformed (SD) case; instead two neutrons occupy the very prolate deformation driving $[440]1/2$ orbits (raising $n_{\frac{1}{2}^+}$ to 10) which is a superposition of $g_{\frac{9}{2}, \frac{7}{2}} d_{\frac{5}{2}, \frac{3}{2}} s_{\frac{1}{2}}$ orbits of $N_{osc} = 4$ origin. In ^{46}Al one $m = \frac{3}{2}^-$ neutron shift to $m = \frac{1}{2}^-$, enhancing the prolate deformation. It is to be emphasized that the deformations of occupied orbits of self-consistent SD solutions are more (than their normal deformed counterparts) because of mixing among the shells.

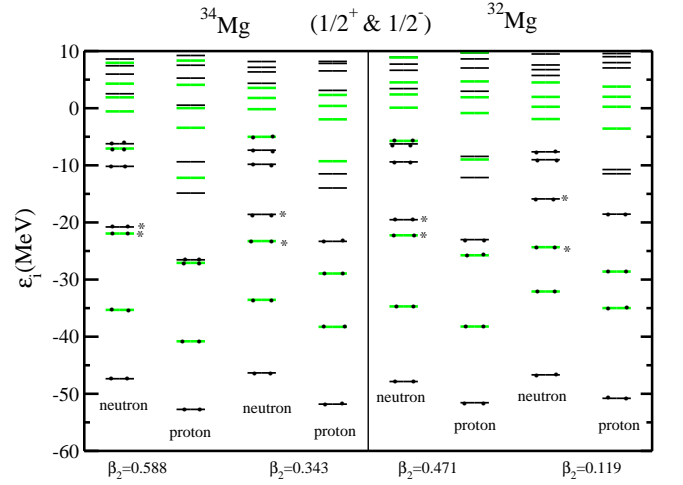


FIG. 5: The $\frac{1}{2}^+$ and $\frac{1}{2}^-$ intrinsic single-particle states for the normal and superdeformed state for ^{32}Mg and ^{34}Mg . Doublets are noticed for the SD intrinsic states only. The $\pm\frac{1}{2}^-$ states are denoted by green lines and the $\pm\frac{1}{2}^+$ states are denoted by black.

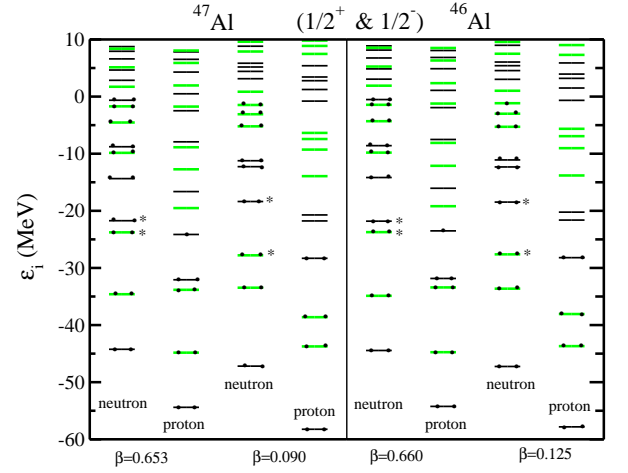


FIG. 6: Same as Fig. 5 for ^{46}Al and ^{47}Al .

1. Structure of Superdeformed Configuration:

We discuss some clear and important characteristics of superdeformed solutions ($\beta \sim 0.5$ or more) obtained in mean field models as compared to the normal solutions of smaller deformation. Since the lowering and occupation of the deformation-driving $\Omega = \frac{1}{2}$ orbits from the shell above the usual valence space is so important in producing superdeformation we have emphasized their role in this discussion. There is the occurrence of $\frac{1}{2}^+$, $\frac{1}{2}^-$ orbits close together in energy (doublets) below and near the Fermi surface of the self-consistent superdeformed solutions. This feature also occurs broadly in Nilsson orbits at asymptotically large prolate deformations (see the Nilsson diagrams in Bohr and Mottelson vol. II [71]).

2. Some features of superdeformed solutions:

In normal deformed case, the deformed orbits of a major shell form a “band”-like set of orbits, distinctly separated from the major shell above and below (see Fig. 6 for ^{47}Al ($\beta=0.09$) and ^{46}Al ($\beta = 0.125$)). Thus physical states obtained from such intrinsic states of low deformation will be well separated in energy from those intrinsic states where excitation occurs across a major shell (a single nucleon excitation across a major shell means a change in parity and significant energy change for small deformation).

The above mentioned “band”-like separation of orbits of major shells of unique parity is quite lost in the case of superdeformation (see Fig. 6, $\beta=0.653$ of ^{47}Al and $\beta=0.660$ of ^{46}Al). The “band”-like orbits now spread in energy (both downward and upward) and orbits of successive major shells come closer to each other in energy; an inter-mingling of orbits of different parities (see Figs. 5, 6). This is a significant structural change from the case of small deformation. This has also been seen in the case of ^{84}Zr in Hartree-Fock study [72, 73].

We would like to emphasize that in the self-consistent models (Skyrme-HF and RMF) the deformation of the nucleus is the result of the deformation of the self-consistently occupied individual orbits:

$$Q = \sum_{i(\text{occupied})} q_i \dots \quad (13)$$

The occupation of the more deformation driving orbits from the shell above the valence space and the unoccupation of oblate driving orbits (e.g. $f_{7/2}$, $m = \pm \frac{7}{2}$) contribute much to configuration mixing and the lowering of $m = \frac{1}{2}$ orbits and to generation of the quadrupole deformation. Because of coming together in energy of $m = \frac{1}{2}^+$ and $\frac{1}{2}^-$ orbits, it is easy to see that superdeformed intrinsic states of two different parities for a particular K quantum number can be formed which will be close to each other in energy. This will lead to parity doublets in band structures. For the neutron-rich nuclei being discussed here, the protons are quite well bound and possible low energy excitations will be those of neutrons near the Fermi surface.

$$|\phi_K\rangle = |\phi_{K_p}^p\rangle > |\phi_{K_n}^n\rangle > \dots, \quad (14)$$

where K_p and K_n are the K quantum numbers for proton and neutron configurations ($K=K_p + K_n$).

3. Examples of parity doublet configurations:

We illustrate schematically possible parity doublet of configurations for neutrons in Fig. 7, the proton configuration $|\phi_{K_p}^p\rangle$ being fixed. We show here the last few neutron occupations of superdeformed solutions and rearrangements near the Fermi surface. In Fig. 7, (b) and (c) are parity doublet of configurations. $A^+ \rightarrow A^-$ transition between (b) and (c) configurations is of odd parity multipole nature.

Thus, in summary, we find a systematic behaviour of the low Ω (particularly $\frac{1}{2}^+$ and $\frac{1}{2}^-$) prolate deformed orbits for

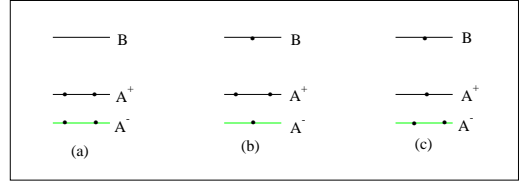


FIG. 7: With parity doublet of occupied orbits A^+ , A^- (having $m = \pm 1/2$ and +ve and -ve parities) and an unoccupied orbits B possible occupation of neutrons are shown in configurations (a), (b) and (c). The two excited configurations (b) and (c) have the same K_n value and represent two excited bands of different parities (parity doublet). Such situation can occur for neutron configurations in superdeformed ^{47}Al and ^{32}Mg , ^{34}Mg (Figs. 6, 5).

the superdeformed solutions. We notice (from the plot of the orbits) that there is occurrence of $\frac{1}{2}^+$ and $\frac{1}{2}^-$ orbits very close to each other in energy for the superdeformed (SD) shape. Such $\frac{1}{2}^+$, $\frac{1}{2}^-$ degenerate orbits occur not only for the well-bound orbits but also for the unbound states. For example, the doublet of neutron orbits $[220]\frac{1}{2}^+$ and $[101]\frac{1}{2}^-$ are 4 MeV apart from each other in the normal deformed prolate solutions; but they become degenerate in the superdeformed (SD) solutions (shown by * in Figs. 5 and 6 for Mg, Al). More such doublets are easily identified (Figs. 5 and 6) for superdeformed solutions of $^{32,34}\text{Mg}$ and $^{46,47}\text{Al}$. In fact it is noticed that the $\Omega = \frac{1}{2}$ states of unique parity, seen clearly well separated in energy from the usual parity orbits in the normal deformed solutions, occurs closer to them in energy for the SD states, showing a degenerate parity doublet structure. In fact, for SD solution the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ orbits are intermixed in the energy plot; while for the normal deformation they occur in distinct groups. This is true both in the Skyrme Hartree Fock and the RMF calculations.

This can be seen by examining the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ orbits for small and large deformations in Fig. 5. This can lead to parity mixing and octupole deformed shapes for the SD structures [72]. Parity doublets and octupole deformation for superdeformed solutions have been discussed for ^{84}Zr [72, 73]. There is much interest for the experimental study of the spectra of neutron-rich nuclei in this mass region [74]. The highly deformed structures for the neutron-rich Ne-Na-Mg-Al nuclei are interesting and signature of such superdeformed configurations (with parity doublet structure) should be looked for.

IV. SUMMARY AND CONCLUSIONS

In summary, we calculate the ground and low-lying excited state properties, like binding energy and quadrupole deformation β_2 using RMF(NL3) formalism for Ne, Na, Mg, Si, P and S isotopes, near the neutron drip-line region. In general, we find large deformed solutions for the neutron-drip nuclei which agree well with the experimental measurement. The calculation is also repeated in the frame-work of nonrelativistic Hartree-Fock formalism with Skyrme interaction SkI4. Both the relativistic and non-relativistic results are compara-

ble to each other for the considered mass region. In the present calculations a large number of low-lying intrinsic superdeformed excited states are predicted in many of the isotopes and some of them are reported. From binding energy point of view, i.e. the sudden fall in S_{2n} value, the breaking of $N = 28$ magic number and the likely appearance of a new magic number at $N = 34$ noticed in our non-relativistic calculations, in contrast with RMF finding. This is an indication of more binding than the neighbouring isotopes. However to confirm $N = 34$ as a magic /non-magic number more calculations are needed. A deformed nucleus has a collective low lying 2^+

state. Also, a spherical nucleus can have a fairly low lying collective 2^+ state (e.g. Sn nuclei) because of quadrupole collectivity. In this study we find that, for the SD shape, the low Ω orbits (particularly $\Omega = \frac{1}{2}$) become more bound and nearly degenerate with the orbits of opposite parity, i.e. they show a parity doublet structure. Closely lying parity-doublet band structures and enhanced odd parity multipole transitions are possible for the superdeformed shapes.

CRP was supported during this work by Project SR/S2/HEP-37/2008 of Department of Science and Technology, Govt. of India.

-
- [1] A. Navin et al., Phys. Rev. Lett. **85**, 266 (2000); H. Iwasaki et al., Phys. Lett. B **491**, 8 (2000); H. Iwasaki et al., Phys. Lett. B **481**, 7 (2000).
 - [2] T. Motobayashi et al., Phys. Lett. B **346**, 9 (1995).
 - [3] K. Tanaka et al., Phys. Rev. Lett. **104**, 062701 (2010).
 - [4] H. Iwasaki et al., Phys. Lett. B **522**, 227 (2001).
 - [5] T. Baumann et al., Nature **449**, 1022 (2007).
 - [6] P. Möller, R. J. Nix, W. D. Myers and W. J. Swiatecki, At. Data and Nucl. Data Tables **59**, 185 (1995).
 - [7] A. Ozawa, T. Kobayashi, T. Suzuki, K. Yoshida and I. Tanihata, Phys. Rev. Lett. **84**, 5493 (2000).
 - [8] T. Minamisono, T. Ohtsubo, I. Minami, S. Fukuda, A. Kitagawa, M. Fukuda, K. Matsuta, Y. Nojiri, S. Takeda, H. Sagawa and H. Kitagawa, Phys. Rev. Lett. **69**, 2058 (1992).
 - [9] I. Tanihata, J. Phys. G **22**, 157 (1996).
 - [10] T. Nakamura, J. Phys. Conf. Ser. **312**, 082006 (2011); T. Nakamura et al., Phys. Rev. Lett. **103**, 262501 (2009).
 - [11] D. Vautherin and D. M. Brink, Phys. Rev. C **5**, 626 (1972).
 - [12] P. -G. Reinhard, Rep. Prog. Phys. **52**, 439 (1989).
 - [13] E. Chabanat, P. Bonche, P. Hansel, J. Meyer and R. Schaeffer, Nucl. Phys. A **627**, 710 (1997).
 - [14] M. Bender, Paul-Henri Heenen and P. -G. Reinhard, Rev. Mod. Phys. **75**, 121 (2003).
 - [15] D. Lunney, J. M. Pearson and C. Thibault, Rev. Mod. Phys. **75**, 1021 (2003).
 - [16] J. R. Stone, J. C. Miller, R. Konciewicz, P. D. Stevenson and M. R. Strayer, Phys. Rev. C **68**, 034324 (2003).
 - [17] J. R. Stone and P. -G. Reinhard, Prog. Part. Nucl. Phys. **58**, 587 (2007).
 - [18] J. Erler, P. Klüpfel and P. -G. Reinhard, J. Phys. G: Nucl. Part. Phys. **38**, 033101 (2011).
 - [19] T. Nakatsukasa, Prog. Theor. Exp. Phys. 01A207 (2012).
 - [20] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
 - [21] Y. K. Gambhir, P. Ring and A. Thimet, Ann. Phys. (N.Y.) **198**, 132 (1990).
 - [22] D. Vretenar, A. V. Afanasjev, G. A. Lalazissis and P. Ring, Physics Reports **409**, 101 (2005).
 - [23] J. Meng et al., Prog. Part. Nucl. Phys. **57**, 470 (2006).
 - [24] T. Nikšić, D. Vretenar and P. Ring, Prog. Part. Nucl. Phys. **66**, 519 (2011).
 - [25] N. Paar, D. Vretenar, E. Khan and Gianluca Coló, Rep. Prog. Phys. **70**, 691 (2007).
 - [26] P. -G. Reinhard and H. F. Flocard, Nucl. Phys. A **584**, 467 (1995).
 - [27] E. Chabanat, P. Bonche, P. Haensel, J. Meyer and R. Schaeffer, Nucl. Phys. A **635**, 231 (1998).
 - [28] M. A. Preston and R. K. Bhaduri, *Structure of Nucleus*, Addison-Wesley Publishing Company, Ch. 8, page 309 (1982).
 - [29] S. K. Patra, Phys. Rev. C **48**, 1449 (1993).
 - [30] J. Decharg and D. Gogny, Phys. Rev. C **21**, 1568 (1980).
 - [31] D. G. Madland and J. R. Nix, Nucl. Phys. A **476**, 1 (1981).
 - [32] S. J. Krieger, P. Bonche, H. Flocard, P. Quentin and M. S. Weiss, Nucl. Phys. A **517**, 275 (1990).
 - [33] H. Zhang et al., Eur. Phys. J. A **30**, 519 (2006).
 - [34] S. Raman, C. W. Jr. Nestor and P. Tikkanen, At. Data and Nucl. Data Tables **78**, 1 (2001).
 - [35] M. Wang, G. Audi, A. H. Wapstra, F. G. Kondev, M. MacCormick, X. Xu and B. Pfeiffer, Chin. Phys. C **36**, 1603 (2012).
 - [36] I. Angeli, K. P. Marinova, At. Data and Nucl. Data Tables **99**, 69 (2013).
 - [37] G. Ripka, Adv. Nucl. Phys. **1**, 183 (1968); W. H. Bassichis, B. Giraud and G. Ripka, Phys. Rev. Lett. **15**, 980 (1965).
 - [38] A. Bohr and B. R. Mottelson, Nuclear Structure Vol. I W A Benjamin, Inc (1969).
 - [39] G. A. Lalazissis, S. Raman and P. Ring, At. Data and Nucl. Data Tables **71**, 1 (1999).
 - [40] S. K. Patra, M. Del. Estel, M. Centelles and X. Viñas, Phys. Rev. C **63**, 024311 (2001).
 - [41] S. K. Patra and C. R. Praharaj, Phys. Rev. C **44**, 2552 (1991).
 - [42] J. Dobaczewski, H. Flocard and J. Treiner, Nucl. Phys. A **422**, 103 (1984).
 - [43] J. Meng and P. Ring, Phys. Rev. Lett. **77**, 3963 (1996).
 - [44] M. Del Estal, M. Centelles, X. Viñas and S. K. Patra, Phys. Rev. C **63**, 044321 (2001).
 - [45] M. Stoitsov, P. Ring, D. Vretenar and G. A. Lalazissis, Phys. Rev. C **58**, 2086 (1998).
 - [46] H. Nakada and M. Sato, Nucl. Phys. A **699**, 511 (2002).
 - [47] Shan-Gui Zhou, J. Meng and P. Ring, Phys. Rev. C **68**, 034323 (2003).
 - [48] Shan-Gui Zhou, J. Meng, P. Ring and En-Guang Zhao, Phys. Rev. C **82**, 011301(R) (2010).
 - [49] P. Arumugam, B. K. Sharma, S. K. Patra and R. K. Gupta, Phys. Rev. C **71**, 064308 (2005).
 - [50] J. Meng and P. Ring, Phys. Rev. Lett. **77**, 3963 (1996).
 - [51] J. P. Elliott and T. H. R. Skyrme, Proc. R. Soc. London A **232**, 561 (1955).
 - [52] J. W. Negele, Phys. Rev. C **1**, 1260 (1970).
 - [53] Y. Sugahara and H. Toki, Nucl. Phys. A **579**, 557 (1994).
 - [54] G. A. Lalazissis, K. König and P. Ring, Phys. Rev. C **55**, 540 (1997).
 - [55] B. A. Brown, Phys. Rev. C **58**, 220 (1998).
 - [56] R. C. Nayak and L. Satpathy, At. Data and Nucl. Data Tables **98**, 616 (2012).
 - [57] P. Möller, J. R. Nix and K. -L. Kratz, At. Data and Nucl. Data Tables **66**, 131 (1997).
 - [58] M. Samyn, S. Goriely, M. Bender and J. M. Pearson, Phys. Rev.

- C **70**, 044309 (2004).
- [59] J. P. Maharana, Y. K. Gambhir, J. A. Sheikh and P. Ring, Phys. Rev. C **46**, R1163 (1992).
- [60] S. K. Patra and C. R. Praharaaj, Phys. Rev. C **47**, 2978 (1993).
- [61] F. Sarazin et al., Phys. Rev. Lett. **84**, 5062 (2000).
- [62] J. L. Egido, L. M. Robledo, R. R. Rodriguez-Guzman, Phys. Rev. Lett. **93**, 282502 (2004).
- [63] N. Onishi and S. Yoshida, Nucl. Phys. **80**, 367 (1966).
- [64] T. K. Jha, M. S. Mehta, S. K. Patra, B. K. Raj and R. K. Gupta, PRAMANA -J. Phys. **61**, 517 (2003); L. Satpathy and S. K. Patra, Nucl. Phys. A **722**, 24c (2003); R. K. Gupta, M. Balasubramaniam, Sushil Kumar, S. K. Patra, G. Münzenberg and W. Greiner, J. Phys. G **32**, 565 (2006); R. K. Gupta, S. K. Patra and W. Greiner, Mod. Phys. Lett. A **12**, 1327 (1997).
- [65] T. R. Werner, J. A. Sheikh, W. Nazarewicz, M. R. Strayer, A. S. Umar and M. Misu, Phys. Lett. B **335**, 259 (1994).
- [66] Ren Zhongzhou, Z. Y. Zhub, Y. H. Cai and Xu Gongou, Phys. Lett. B **380**, 241 (1994).
- [67] I. Hamamoto, Phys. Rev. C **76**, 054319 (2007); *ibid* C **85**, 064329 (2012).
- [68] R. K. Gupta, S. K. Patra and W. Greiner, Mod. Phys. Lett. A **12**, 1317 (1997).
- [69] M. M. Sharma, M. A. Nagarajan and P. Ring, Phys. Lett. B **312**, 377 (1993).
- [70] R. Kanungo, I. Tanihata and A. Ozawa, Phys. Lett. B **528**, 58 (2002).
- [71] A. Bohr and B. R. Mottelson, Nuclear Structure, vol. II (, Chapter 5
- [72] C. R. Praharaaj, *INT Workshop on "Nuclear Many-Body Approaches for the 21st Century"*, September 24 - November 30, 2007, Institute for Nuclear Theory website, University of Washington, Seattle.
- [73] C. R. Praharaaj, *Structure of Atomic Nuclei*, Ch. 4, page 108, Edited by Satpathy L, Narosa Publishers (Delhi) (1999).
- [74] D. Miller et al., Phys. Rev. C **79**, 054306 (2009).